Real Numbers

EXERCISE 1.1

1) Use Euclid's division algorithm to find the HCF of:
   i) 135 and 225
   ii) 196 and 38220
   iii) 867 and 255

Answer:

i. 225 > 135 we always divide greater number with smaller one.
Divide 225 by 135 we get 1 quotient and 90 as remainder so that 225 = 135 x 1 +90
Divide 135 by 90 we get 1 quotient and 45 as remainder so that 135 = 90 x 1 +45
Divide 90 by 45 we get 2 quotient and no remainder so we can write it as 90 = 2 x 45 + 0
As there is no remainder so divisor 45 is our HCF.

ii. 38220 > 196 we always divide greater number with smaller one.
Divide 38220 by 196 then we get quotient 195 and no remainders so we can write it as 38220 = 196 x 195 + 0
As there is no remainder so divisor 196 is our HCF.

iii. 867 > 255 we always divide greater number with smaller one.
Divide 867 by 255 then we get quotient 3 and remainder is 102 so we can write it as 867 = 255 x 3 + 102
Divide 255 by 102 then we get quotient 2 and remainder is 51 so we can write it as 255 = 102 x 2 + 51
Divide 102 by 51 we get quotient 2 and no remainder so we can write it as 102 = 51 x 2 + 0
As there is no remainder so divisor 51 is our HCF.

2) Show that any positive integer odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5 , where q is some integer.

Answer:

Lets take 'a' as as any positive integer and b = 6.
Then using Euclids algorithm we get a = 6q +r here r is a remainder and value of q is more than or equal to 0 and r = 0 , 1 , 2 , 3 , 4 , 5 because $0 \leq r < b$ and the value of b = 6.
So total possible forms will be 6q+0 , 6q + 1, 6q + 2 , 6q + 3 , 6q+4 , 6q+5

Case 1: 6q+0
6 is divisible by 2 so it is an even number

Case 2: 6q+1
6 is divisible by 2 but 1 is not divisible by 2, so it is an odd number

Case 3: 6q+2
6 is divisible by 2 and 2 is also divisible by 2, so it is even number

Case 4: 6q+3
6 is divisible by 2 but 3 is not divisible by 2, so it is an odd number

Case 5: 6q+4
6 is divisible by 2 and 4 is also divisible by 2, so it is even number

3) An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer:

HCF (616, 32) will give the maximum number of columns in which they can march.
We can use Euclid's algorithm to find the HCF.
616 = 32 x 19 + 8
32 = 8 x 4 + 0
The HCF (616, 32) is 8.
Therefore, they can march in 8 columns each.

4) Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.
[Hint: Let x be any positive integer then it is of the form 3q, 3q+1 or 3q +2. Now square each of these and show that they can be rewritten in the form 3m or 3m+1.]
Answer:

Let "a" be any positive integer and b = 3. Then a = 3q +r for some integer $q \geq 0$. And r = 0, 1, 2 because $0 \leq r < 3$.
Therefore, a = 3q or 3q + 1 or 3q + 2
or,
$a^2 = (3q)^2 \; or \; (3q+1)^2 \; or \; (3q+2)^2$
$a^2 = (9q)^2 \; or \; 9q^2 + 6q + 1 \; or \; 9q^2 + 12q + 4$
= $3 \times (3q)^2 \; or \; 3(3q^2 + 2q) + 1 \; or \; 3(3q^2 + 4q + 1)$

= $3k_1$ or $3k_2 + 1$ or $3k_3 + 1$

Where k_1, k_2 and k_3 are some positive integers . Hence, it can be said that the square of any positive integer is either of the form 3m or 3m+1.

5) Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m+1 or 9m + 8.

Answer:

Let a be any positive integer and b=3
a = 3q + r , where $q \geq 0$ and $0 \leq r < 3$
Therefore, a = 3q or 3q+1 or 3q+2.

There are three cases.

Case 1: When a = 3q,
$a^3 = (3q)^3 = 27q^3 = 9(3q)^3 = 9m,$
Where m is an integer such that m = $3q^3$

Case 2: When a = 3q +1,
$a^3 = (3q + 1)^3$
$a^3 = 27\,q^3 + 27q^2 + 9q + 1$
$a^3 = 9(3q^3 + 3q^2 + q) + 1$
$a^3 = 9m + 1$
Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When a = 3q +2,
$a^3 = (3q + 2)^3$
$a^3 = 27\,q^3 + 54q^2 + 36q + 8$
$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$
$a^3 = 9m + 8$
Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$. Therefore, the cube of any positive integer is of the form 9m, 9m + 1, or 9m + 8.