Real Numbers

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EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

I. $\frac{13}{3125}$

II. $\frac{17}{8}$

III. $\frac{64}{455}$

IV. $\frac{15}{1600}$

V. $\frac{29}{343}$

VI. $\frac{23}{2^3 5^2}$

VII. $\frac{129}{2^2 5^7 7^5}$

VIII. $\frac{6}{15}$

IX. $\frac{35}{50}$

X. $\frac{77}{210}$

Answer:

I. $\frac{13}{3125}$

$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

II. $\frac{17}{8}$

$8 = 2 \times 2 \times 2 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

III. $\frac{64}{455}$

$455 = 4 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non – terminating repeating.

IV. $\frac{15}{1600}$

1600 = 2 x 2 x 2 x 2 x 2 x 2 x 5 x 5 = $2^6 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

V. $\frac{29}{343}$

343 = 7 x 7 x 7 = 7^3

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion $\frac{29}{343}$ is non-terminating.

VI. $\frac{23}{2^3 5^2}$

Denominator = $2^3 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 5^2}$ is terminating.

VII. $\frac{129}{2^2 5^7 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

VIII. $\frac{6}{15}$

$$\frac{6}{15} = \frac{2 * 3}{5 * 3} = \frac{2}{5}$$

The denominator is of the form 5^n

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

IX. $\frac{35}{50}$

$$\frac{35}{50} = \frac{5 * 7}{2 * 5 * 5} = \frac{7}{10}$$

Denominator = 10 = 2 x 5

The denominator is of the form $2^m \times 5^n$. Hence the decimal expansion of $\frac{35}{50}$ is terminating.

X. $\frac{77}{210}$

$$\frac{77}{210} = \frac{7 * 11}{2 * 3 * 5 * 7} = \frac{11}{30}$$

Denominator = 30 = 2 x 3 x 5

Since the denominator is not of the form $2^m \times 5^{n,}$ and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

2. Write down the decimal expansions of those rational numbers in Question-1 above which have terminating decimal expansions.

Answer:

I. $\frac{13}{3125}$

$$\frac{13}{3125} = \frac{13}{5*5*5*5*5} = \frac{13}{5^5} * \frac{2^5}{2^5} = \frac{13*32}{(5*2)^5} = \frac{416}{10^5} = 0.00416$$

II. $\frac{17}{8}$

$$\frac{17}{8} = \frac{17}{2*2*2} = \frac{17}{2^3} * \frac{5^3}{5^3} = \frac{17*125}{(2*5)^3} = \frac{2125}{10^3} = 2.125$$

III. $\frac{64}{455}$

Decimal expansion is non-terminating repeating.

IV. $\frac{15}{1600}$

$\frac{15}{1600} = \frac{3*5}{2*2*2*2*2*2*5} = \frac{3}{2^6*5} X \frac{5^5}{5^5} = \frac{3*3125}{(2*5)^6} = \frac{9375}{10^6} = 0.009375$

V. $\frac{29}{343}$

Decimal expansion is non-terminating repeating.

VI. $\frac{23}{2^3 5^2}$

$$\frac{23}{2^3 * 5^2} X \frac{5}{5} = \frac{(23*5)}{(2*5)^3} = \frac{115}{(10)^3} = 0.115$$

VII. $\frac{129}{2^2 5^7 7^5}$

Decimal expansion is non-terminating repeating.

VIII. $\frac{6}{15}$

$$\frac{6}{15} = \frac{2*3}{3*5} = \frac{2}{5} X \frac{2}{2} = \frac{2*2}{2*5} = \frac{4}{10} = 0.4$$

IX. $\frac{35}{50}$

$$\frac{35}{50} = \frac{5*7}{2*5*5} = \frac{7}{2*5} = \frac{7}{10} = 0.7$$

X. $\frac{77}{210}$

Decimal expansion is non-terminating repeating.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

I. 43.12456789
II. 0.120120012000120000…
III. 43.123456789

Answer:

I. 43.12456789
Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$ i.e. the prime factors of q will be either 2 or 5 or both.

II. 0.120120012000120000…
The decimal expansion is neither terminating nor recurring. Therefore, the given number is irrational.

III. 43.123456789
Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e. the prime factors of q will also have a factor other than 2 or 5.